Restricted Draft Report **SR.15.BinnedParetoMLE**

Maximum Likelihood Estimates of *b*-Value for Induced Seismicity in the Groningen Field

by

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Executive summary

Studies of earthquake catalogues for naturally occurring seismicity led to the formulation of the Gutenberg-Richter law [1] which states that the probability $P(m)$ that an observed event in a sequence of earthquakes has a moment magnitude greater than m satisfies the relation $P(m) = \exp\{-b\ln(10)(m - m_0)\}$, where m_0 is the observation threshold. Evidence has emerged since the formulation of the law that the same relation applies for induced seismicity caused by human activity, an observation, together with the power-law character of the law when translated into moments rather than moment magnitudes, that the Earth's crust may be in a self-organized critical state [2]. The use of a robust method for estimating the value of b from seismic data is crucial for quantifying the risk of rare but powerful events. In the case of the Groningen field, seismic activity due to gas production started in 1991 and has continued ever since. Moment magnitude data is provided by KNMI (the Dutch Meteorological Institute) and is rounded to the nearest 0.1. This means that the data has to be regarded as being binned, with the bin boundaries located at odd multiples of 0.05.

In this report the maximum likelihood estimate is applied to this binned data and a maximum likelihood estimator b_{MLE} is obtained for the whole catalogue, and various temporal and spatial subsets. We remark that only events with magnitude greater or equal to 1.5 and occurring after the end of April 1995 are considered in this analysis, as only for such events can reliable detection be assured [3]. The fact that the data bins are of equal width in moment-magnitude space means that b_{MLE} can be determined analytically. Monte-Carlo simulations of synthetic catalogues are used to determine estimates of the error in b_{MLE}.

The main conclusions of this study are the following:

1. The value of b_{MLE} for the entire Groningen catalogue (1^{st} January 1995 – 31^{st} December 2014) is consistent with the commonly accepted value $b = 1$.
2. While a subdivision of the catalogue into four subsets of equal numbers of events suggests more variability in the maximum likelihood estimator for the subsets than would be expected if the b-value of the underlying process were constant, a closer examination, involving further subdivisions of the catalogue, and the use of 95% confidence intervals obtained using simulated data, does not show any systematic dependence of b value on event rate.
3. Focusing on regions of 5 km radius centered on Loppersum and Ten Boer, we find that, while the b_{MLE} for Ten Boer is consistent with $b = 1$, the b_{MLE} for Loppersum is significantly lower, to the extent that we can be 99.9% confident that the b value of the underlying process is less than 1.
4. The temporal sequence of events for each of the regions around Loppersum and Ten Boer is split into two equal sub-catalogues and the value of b_{MLE} determined for each sub-catalogue. For both the Loppersum and Ten Boer regions, the two values of b_{MLE} were found to be consistent with an underlying b that is unchanging in time.

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1. Introduction

The moment magnitude *m* is a logarithmic scale unit used to describe seismic moment *M* and has no physical significance in itself. Other examples of logarithmic scale units are decibels for acoustic energy, stellar magnitude for star brightness and gauge for wire diameter. The moment magnitude *m* corresponding to a seismic moment *M* is given by [4]

$$m = \{\log_{10}(M / M_c)\}/ d \tag{1.1}$$

In Eq. (1.1), *d* is a dimensionless constant with the value 1.5 and $M_c = 1.259 \times 10^9$ N-m. We note that Eq. (1.1) can also be written as

$$m = \{\ln(M / M_c)\}/\{d \ln(10)\} \tag{1.2}$$

According to the Gutenberg-Richter law, the probability *P*(*m*) that an observed event in a sequence of earthquakes has a moment magnitude greater than *m* satisfies the relation

$$P(m) = \exp\{-b \ln(10)(m - m_0)\} \tag{1.3}$$

where m_0 is the observation threshold. If M_0 is the threshold seismic moment corresponding to m_0, we have, from Eq. (1.2),

$$m_0 = \{\ln(M_0 / M_c)\}/\{d \ln(10)\} \tag{1.4}$$

Subtracting Eq. (1.4) from Eq. (1.2), and rearranging the right-hand side yields

$$m - m_0 = \{\ln(M / M_0)\}/\{d \ln(10)\} \tag{1.5}$$

Substituting Eq. (1.5) into Eq. (1.3) yields the result

$$F(M) = P(m) = (M / M_0)^{-\beta} \tag{1.6}$$

where $\beta = b / d$ (1.7)

and *F*(*M*) denotes the probability that the seismic moment of an observed event is greater than *M*. For induced seismicity, a commonly accepted value of *b* is unity, corresponding to a β value of 2/3. The distribution *F*(*M*) is a long-tailed Pareto distribution and has infinite variance for $\beta \leq 2$ and infinite mean for $\beta \leq 1$. Since the threshold for reliable detection of a seismic event can be specified, *F*(*M*) can be regarded as a one-parameter distribution with parameter β. We also note that Eq. (1.6) allows for *M* to be unbounded, which is unphysical. For the Groningen field, upper bounds on the maximum possible value of M have been specified [3], but such an event is so rare that the maximum moment of an observed event in the Groningen catalogue to date is far below it. Hence the existence of a "corner moment", at which $F(M)$ starts to reduce at a rate faster than the long tail in Eq. (1.6), cannot be inferred from the data. The focus of this report is the estimation of the parameter β from the observed data (which directly gives b via Eq. 1.7). This is done both for the entire catalogue, and also for temporal and spatial subsets.

Chapter 2 describes the methodology used, which employs the maximum likelihood estimator, and shows that this quantity can be evaluated analytically. Chapter 2 also addresses the estimation of error in the estimate, which is obtained using simulated catalogues. Other methods of obtaining this error, and also the bias in the maximum likelihood estimator, are outlined in the appendix.

The results are presented in Chapter 3, which contains estimates of b value for the entire catalogue, temporal subsets of it, and for two subregions centered around Loppersum and Ten Boer. Chapter 4 contains the conclusions and recommendations for future work.

2. Maximum Likelihood Estimator for Binned Seismic Data

As mentioned in the introduction, we take magnitude 1.5 to be the threshold of reliable observation. Since earthquake magnitudes are rounded to the nearest 1.5, this corresponds to a bin spanning a range of magnitudes 1.45 to 1.55. Multiplying Eq. (1.5) by $d\ln(10)$, exponentiating and rearranging, we obtain

$$M_0 = M_c 10^{dm_0} \tag{2.1}$$

Setting m_0 =1.45, d = 1.5 and M_c = 1.259 × 10^9 N-m in Eq. (2.1), we obtain

$$M_0 = 1.884 \times 10^{11} \text{ N-m} \tag{2.2}$$

We shall work with the normalized moment

$$x = M / M_0 \tag{2.3}$$

This means that the first bin for x starts at x = 1 and ends at α, where

$$\alpha = 10^{0.1d} = 10^{0.15} = 1.413 \tag{2.4}$$

The kth bin runs from $x = \alpha^{k-1}$ to α^k. If the nth bin is the last non-empty bin, all the empty bins following it are included in the nth bin, so that it extends from $x = \alpha^{n-1}$ to ∞. The probability p_k that an event falls within the nth bin is given by

$$p_k = F_C(\alpha^{k-1}) - F_C(\alpha^k);\ k = 1, 2 \ldots n-1 \tag{2.5a}$$

$$p_n = F_C(\alpha^{n-1}) \tag{2.5b}$$

where (*cf* Eq. 1.6)

$$F_C(x) = x^{-\beta} \tag{2.6}$$

It follows that

$$p_k = \alpha^{-\beta(k-1)} - \alpha^{-\beta k};\ k = 1, 2 \ldots n-1 \tag{2.7a}$$

$$p_n = \alpha^{-\beta(n-1)} \tag{2.7b}$$

For a catalogue containing N events, the probability that bins 1, 2 to n contain r_1, r_2 to r_n events is given by

$$P(r_1, r_2 ... r_n) = N! \prod_{k=1}^{n} \left\{ p_k^{\;r_k} / r_k! \right\} \tag{2.8}$$

where $\sum_{k=1}^{n} r_k = N$ (2.9)

If we are given the r_k and wish to make inferences about the p_k, the right-hand side of Eq. (2.8) can be expressed as a likelihood function $L(p_1, p_2 \ldots p_n)$. That is

$$L(p_1, p_2 ... p_n) = N! \prod_{k=1}^{n} \left\{ p_k^{\;r_k} / r_k! \right\} \tag{2.10}$$

According to the maximum likelihood method, the best value of the parameter β controlling the p_k that fits the observations (the values of r_k) is the one that maximizes $L(p_1, p_2 \ldots p_n)$. Assuming that L is a smooth function of β, β maximizes L locally if

$$\frac{\partial L}{\partial \beta} = 0 \tag{2.11}$$

and $$\frac{\partial^2 L}{\partial \beta^2} < 0 \tag{2.12}$$

We now show that the value of β satisfying Eq. (2.11) can be determined analytically when the p_k are given by Dividing Eq. (2.11) by L we may write it as

$$\frac{\partial}{\partial \beta} \ln(L) = 0 \tag{2.13}$$

Substituting Eq. (2.10) into Eq. (2.13) we deduce that

$$\sum_{k=1}^{n} r_k \frac{\partial}{\partial \beta} \ln(p_k) = 0 \tag{2.14}$$

Using Eq. (2.7a) we have

$$\ln(p_k) = -\beta k \ln(\alpha) + \ln(\alpha^{\beta} - 1) \text{; } k = 1, 2 \ldots n-1 \tag{2.15a}$$

$$\ln(p_n) = -\beta(n-1)\ln(\alpha) \tag{2.15b}$$

so that
$$-\left\{\sum_{k=1}^{n} kr_k - r_n\right\}\ln(\alpha) + (N - r_n)\ln(\alpha)\alpha^{\beta}/(\alpha^{\beta}-1) = 0 \tag{2.16}$$

Eq. (2.16), and hence Eq. (2.11) is satisfied for

$$1-\alpha^{-\beta} = \frac{N - r_n}{\sum_{k=1}^{n} kr_k - r_n} \tag{2.17}$$

This can be written as

$$\alpha^{-\beta} = \frac{\sum_{k=1}^{n}(k-1)r_k}{\sum_{k=1}^{n} kr_k - r_n} \tag{2.18}$$

Hence the value of β satisfying Eq. (2.18), which we henceforth denote by β_{MLE}, is given by

$$\beta_{MLE} = \frac{\ln\left\{\sum_{k=1}^{n} kr_k - r_n\right\} - \ln\left\{\sum_{k=1}^{n}(k-1)r_k\right\}}{\ln(\alpha)} \tag{2.19}$$

Note that β_{MLE} is infinite in the special case that $r_1 = N$ and $r_2 = r_3 = \ldots r_N = 0$. Hence we exclude this case when evaluating the statistics of β_{MLE}. Let $\sum_{\{r_1,\ldots r_n\}}^{N} f(r_1, r_2 \ldots r_n; \varepsilon)$ denote the sum of a function $f(r_1, r_2 \ldots r_n; \varepsilon)$ over all non-negative integers $r_1, r_2 \ldots r_n$ such that $\sum_{k=1}^{n} r_k = N$ and let $\sum_{\{r_1,\ldots r_n\}}^{N}{}'$ denote the same sum, but with the restriction that $r_2, r_3 \ldots r_n$ are not all zero. We consider the restricted average of β_{MLE} given by

$$\mu_{MLE} = <\beta_{MLE}>' = N! \sum_{\{r_1,\ldots r_n\}}^{N}{}' \prod_{k=1}^{n} (p_k^{\,r_k}/r_k!) \sum_{\{r_1,\ldots r_n\}}^{N} \beta_{MLE}/(1-p_1^{\,N}) \tag{2.20}$$

Similarly, the variance of β_{MLE} is given by

$$\sigma_{MLE} = <\beta_{MLE}^{\ 2}>' - <\beta_{MLE}>'^{2} \tag{2.21}$$

where $$<\beta_{MLE}{}^2>' = N!\sum_{\{r_1,\ldots r_n\}}^{N}{}'\prod_{k=1}^{n}(p_k{}^{r_k}/r_k!)\sum_{\{r_1,\ldots r_n\}}^{N}\beta_{MLE}{}^2/(1-p_1{}^N) \tag{2.22}$$

In this report we estimate μ_{MLE} and σ_{MLE} by performing Monte-Carlo simulations of catalogues with a specified β. If u is a random number uniformly distributed between 0 and 1, representing $F(x)$, then Eq. (2.6) may be used to obtain x:

$$x = (1-u)^{-1/\beta} \tag{2.23}$$

The value of 1 + INT{ln(x)/ln(α)}, where INT denotes integer part, then determines the number of the bin in which x is located.

As we shall see in the next chapter 1000 realizations of a catalogue is sufficient for confidence testing and error estimation. An alternative method of obtaining μ_{MLE} and σ_{MLE}, based on transforming the summations on the right-hand sides of Eqs. (2.20) and (2.21) into integrations over one or two variables, is given in the appendix. These integrals may subsequently be evaluated numerically or used to obtain large-N asymptotics.

3. Results

3.1 Maximum-Likelihood Estimate for the *b*-Value of the Entire Groningen Catalogue and its Error Bounds

The Groningen catalogue since 1st May 1995 and up to the end of 2014 comprises 236 events of moment magnitude 1.5 or greater. The maximum observed magnitude was 3.6, observed for a single event. The seismic moments for the catalogue are therefore divided up into 22 bins, corresponding to moment magnitudes of 1.45-1.55, 1.55-1.65 and so forth, up to 3.45-3.55 and finally 3.55-∞. The number of events corresponding to each bin is tabulated in Table 3.1.

Table 3.1. Distribution of Events in Seismic Moment Bins for the 236-Event Groningen Catalogue (1st May 1995-31st December 2014, Rounded Moment Magnitude ≥ 1.5.

Bin Number	1	2	3	4	5	6	7	8
Moment Magnitude	1.5	1.6	1.7	1.8	1.9	2	2.1	2.2
Number of Events	54	38	26	23	18	15	7	8
Bin Number	9	10	11	12	13	14	15	16
Moment Magnitude	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3
Number of Events	8	8	8	5	4	3	1	5
Bin Number	17	18	19	20	21	22		
Moment Magnitude	3.1	3.2	3.3	3.4	3.5	3.6+		
Number of Events	0	3	0	0	1	1		

The maximum likelihood estimate of the β-value for this catalogue, according to Eq. (2.19) is

$$\beta_{MLE} = 0.644 \tag{3.1}$$

This corresponds to a maximum likelihood estimate of *b*-value given by

$$b_{MLE} = 1.5 \times 0.644 = 0.966 \tag{3.2}$$

This is so close to the accepted value $b = 1$ for induced seismicity that we use the value $b = 1$, corresponding to $\beta = 2/3$, to generate 1000 synthetic Monte-Carlo realizations of the 236-event Groningen catalogue, in order to obtain an error estimate for Eqs. (3.1) and (3.2). Figure 3.1 shows the resulting distribution of β_{MLE}, which also indicates the simulation mean (0.664) and the actual catalogue value of β_{MLE} given in Eq. (3.1). The actual catalogue value is sufficiently close to the simulation mean for the hypothesis that $\beta = 2/3$ to be accepted, which means that we can use the standard deviation of the simulations to provide an error bar on Eq. (3.1). This error bar is ± 0.043. Hence we may write

$$\beta_{MLE} = 0.644 \pm 0.043 \qquad (3.3)$$

The corresponding result for b_{MLE} is

$$b_{MLE} = 0.966 \pm 0.065 \qquad (3.4)$$

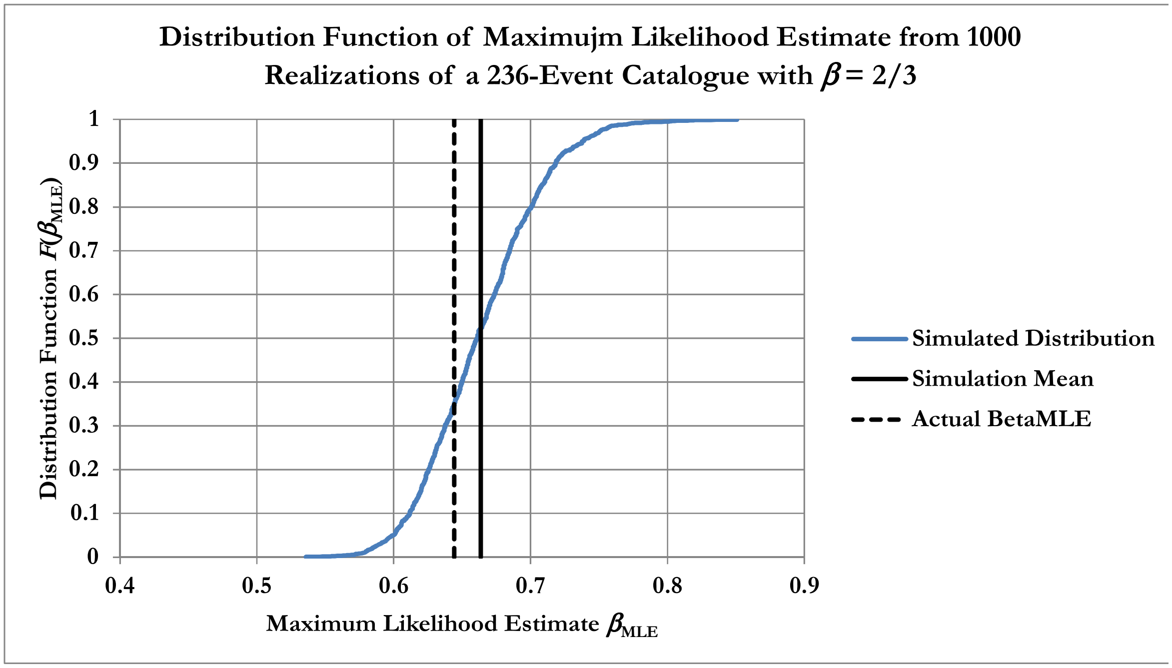


Figure 3.1. Distribution Function of β_{MLE} obtained from 1000 Realizations of a 236-Event Catalogue with $\beta = 2/3$. The vertical solid line shows the value of the simulation mean while the dashed line is the actual Groningen Catalogue β_{MLE}.

The simulation is of value in providing further information about the statistics of the event catalogues. For example, a lower β value results in a greater probability of higher magnitude events, so we can ask the question about how well the event, or events, of maximum magnitude in a simulated catalogue correlate with the β_{MLE} for that catalogue. The results are shown in Figure 3.2. The highest magnitude event, a 6.9, occurs for a β_{MLE} of 0.629, only a little lower than the simulation mean, while of the three catalogues for which the highest magnitude event is only magnitude 3, two of them have high values of β_{MLE}, 0.747 and 0.750, while the remaining one has a β_{MLE} of only 0.612. The complementary distribution function for the maximum magnitude event, or events, in a catalogue, is shown in Figure 3.3. We take note of the fact that 79% of the simulated catalogues contain an event of maximum magnitude greater than 3.6, the observed value for the actual catalogue. The shape of the curve in Figure 3.3 may be readily understood by considering the probability $P_N(x)$ that at least one event out of N has a moment magnitude greater than x. This is given by

$$P_N(x) = 1 - \{1 - F_C(x)\}^N \quad (3.5)$$

where $F_C(x)$ is given by Eq. (2.6). The probability that at least one event has moment magnitude greater than m, $P_N(m)$, is then given by

$$P_N(m) = P_N(x) \quad (3.6)$$

where $x = 10^{(m-1.45)}$ (3.7)

Substituting Eq. (3.6) into Eq. (3.5), and applying Eqs. (3.7) and (2.6), we obtain

$$P_N(m) = 1 - \{1 - 10^{-\beta(m-1.45)}\}^N \quad (3.8)$$

Eq. (3.8) is also included in Figure 3.3 and is in close agreement.

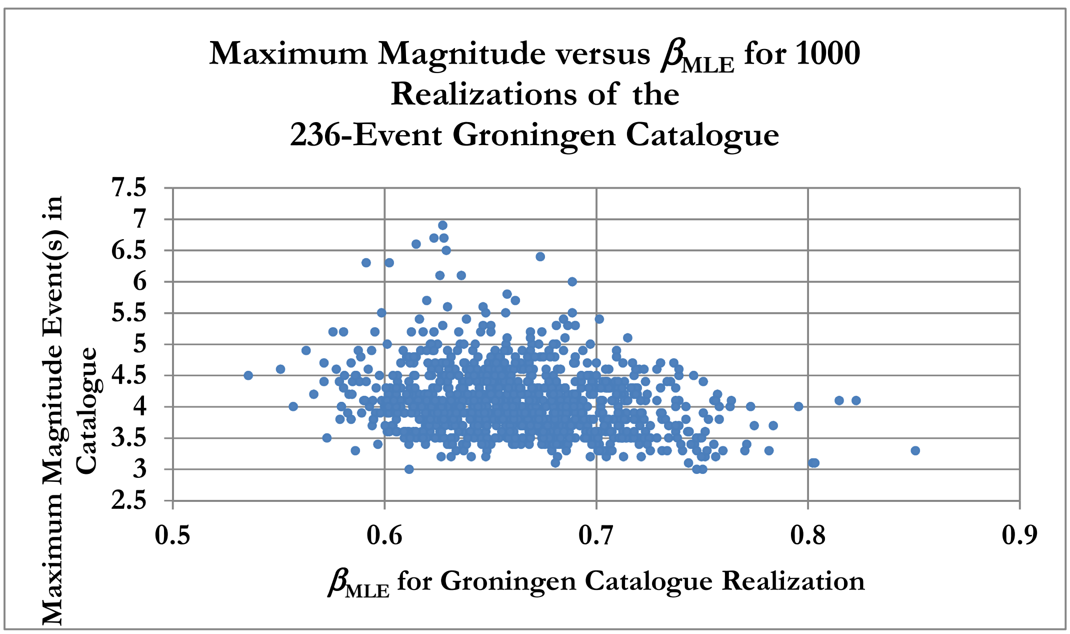


Figure 3.2. Maximum magnitude event(s) in simulated catalogue versus β_{MLE}.

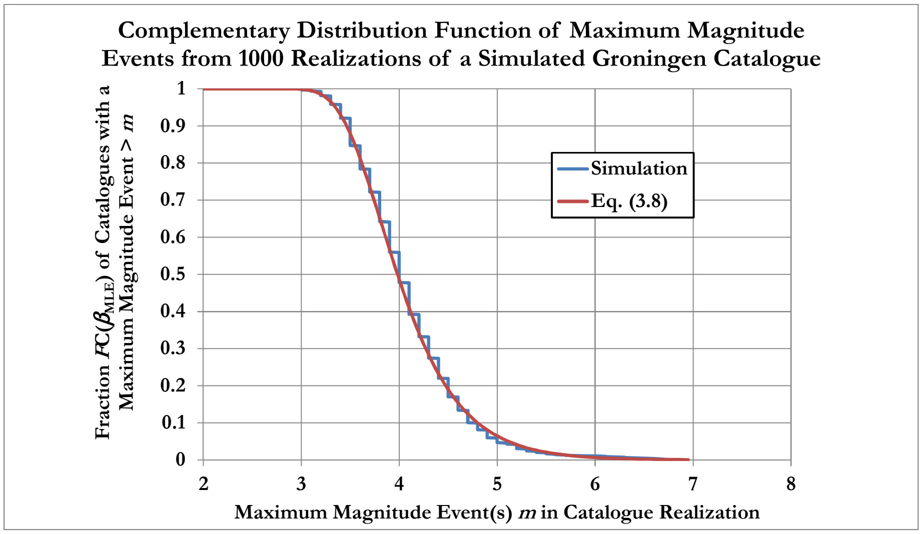


Figure 3.3. *Complementary Distribution Function of Maximum Magnitude Events from 1000 Realizations of a 236-Event Groningen Catalogue. The Red Curve is the Analytical Expression in Eq. (3.8)*

3.2 Maximum Likelihood and Error Estimates for the *b*-Value of Temporal Subsets of the Groningen Catalogue

In this section the Groningen catalogue is divided into four sub-catalogues, each containing 59 events, according to temporal order.

Table 3.2. Distribution of Events in Seismic Moment Bins for the 236-Event Groningen Catalogue, Split Up by Temporal Sub-catalogue

Bin Number	1	2	3	4	5	6	7	8
Moment Magnitude	1.5	1.6	1.7	1.8	1.9	2	2.1	2.2
# Events 1st Subcatalogue	11	16	6	6	3	3	2	2
# Events 2nd Subcatalogue	13	6	7	5	5	3	2	5
# Events 3rd Subcatalogue	16	10	8	7	5	3	0	1
# Events 4th Subcatalogue	14	6	5	5	5	6	3	0
Bin Number	9	10	11	12	13	14	15	16
Moment Magnitude	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3
# Events 1st Subcatalogue	1	2	2	1	2	0	0	2
# Events 2nd Subcatalogue	4	1	2	3	0	0	0	1
# Events 3rd Subcatalogue	2	2	4	0	0	0	0	0
# Events 4th Subcatalogue	1	3	0	1	2	3	1	2
Bin Number	17	18	19	20	21	22		
Moment Magnitude	3.1	3.2	3.3	3.4	3.5	3.6+		
# Events 1st Subcatalogue	0	0	0	0	0	0		
# Events 2nd Subcatalogue	0	1	0	0	1	0		
# Events 3rd Subcatalogue	0	1	0	0	0	0		
# Events 4th Subcatalogue	0	1	0	0	0	1		

The maximum likelihood estimates of the β-value for each sub-catalogue, according to Eq. (2.18) are (with corresponding *b*-value maximum likelihood in brackets):

1st sub-catalogue $$\beta_{MLE} = 0.695 \; (b_{MLE} = 1.043) \tag{3.5a}$$

2nd sub-catalogue $$\beta_{MLE} = 0.577 \; (b_{MLE} = 0.865) \tag{3.5b}$$

3rd sub-catalogue $$\beta_{MLE} = 0.809 \; (b_{MLE} = 1.213) \tag{3.5c}$$

4th sub-catalogue $$\beta_{MLE} = 0.528 \; (b_{MLE} = 0.782) \tag{3.5d}$$

A simulation of 1000 realizations of a 59-event catalogue with $\beta = 2/3$ yields the result

$$\beta_{MLE} = 0.665 \pm 0.086 \tag{3.6}$$

Inspection of Eqs. (3.5) reveals that all the sub-catalogues with the exception of the first lie outside the error bars in Eq. (3.6). This is represented graphically in Figure 3.4, where the values of β_{MLE} given in Eqs. (3.5) are compared with the distribution of β_{MLE} obtained from the simulation.

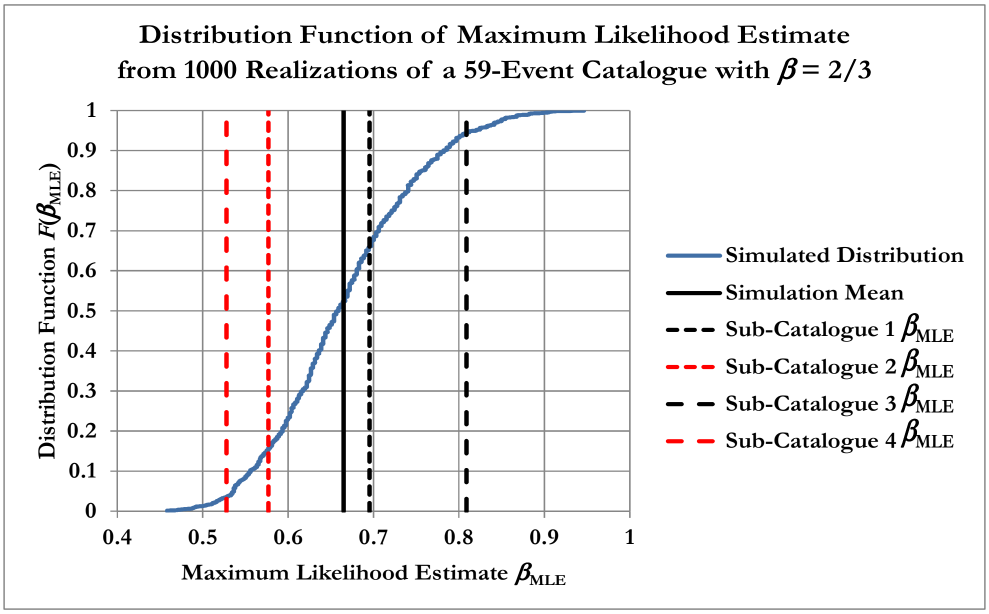


Figure 3.4. Distribution Function of Maximum Likelihood Estimator Obtained from 1000 Realizations of a 59-Event Catalogue with $\beta = 2/3$. The Simulation Mean, Together with the Four Values of β_{MLE} Given in Eqs. 93.45), are Displayed as Vertical Lines on the Plot.

We see from Figure 3.4 that the hypothesis that the β value for the fourth sub-catalogue is less than 2/3 has a confidence level greater than 95%, and that we can be almost as confident that the β value for the third sub-catalogue is greater than 2/3. Note that the probability that at least two of the sub-catalogues have β_{MLE} values in the bottom or top 5% of the distribution is $1 - 0.9^4 - 4 \times 0.1 \times 0.9^3 = 0.0523$. We now determine whether there is any correlation with rate effect by examining the time intervals corresponding to the four sub-catalogues. The dates and times of the first and last events in each sub-catalogue are listed in Table 3.3. The maximum likelihood estimate for each sub-catalogue is plotted against the catalogue duration in Figure 3.5. It is hard to draw any conclusions regarding rate dependency from Figure 3.4, and we further subdivide the catalogue into 1a, 1b, 2a, 2b *etc.* where the *a*-half of each of the four sub-catalogues consists of 29 events and the *b*-half 30 events. The number of events by magnitude for each of these sub-catalogues is listed in Table 3.4.

Table 3.3 Dates and Times of First and Last Events for Each of the Four 59-Event Temporal Sub-Catalogues of the 236 Event Groningen Catalogue

	Date	Time	Date	Time	Elapsed Hrs.
1st Sub-Catalogue	15/05/1995	08:24:00	25/02/2005	20:38:24	3586.24
2nd Sub-Catalogue	10/03/2005	18:43:12	08/05/2009	22:48:00	1524.08
3rd Sub-Catalogue	05/07/2009	08:52:48	19/02/2012	02:52:48	953.00
4th Sub-Catalogue	31/03/2012	09:21:36	30/12/2014	**Not Recorded**	1006.64*

*Time of last event for fourth sub-catalogue is assumed to be 12:00 pm

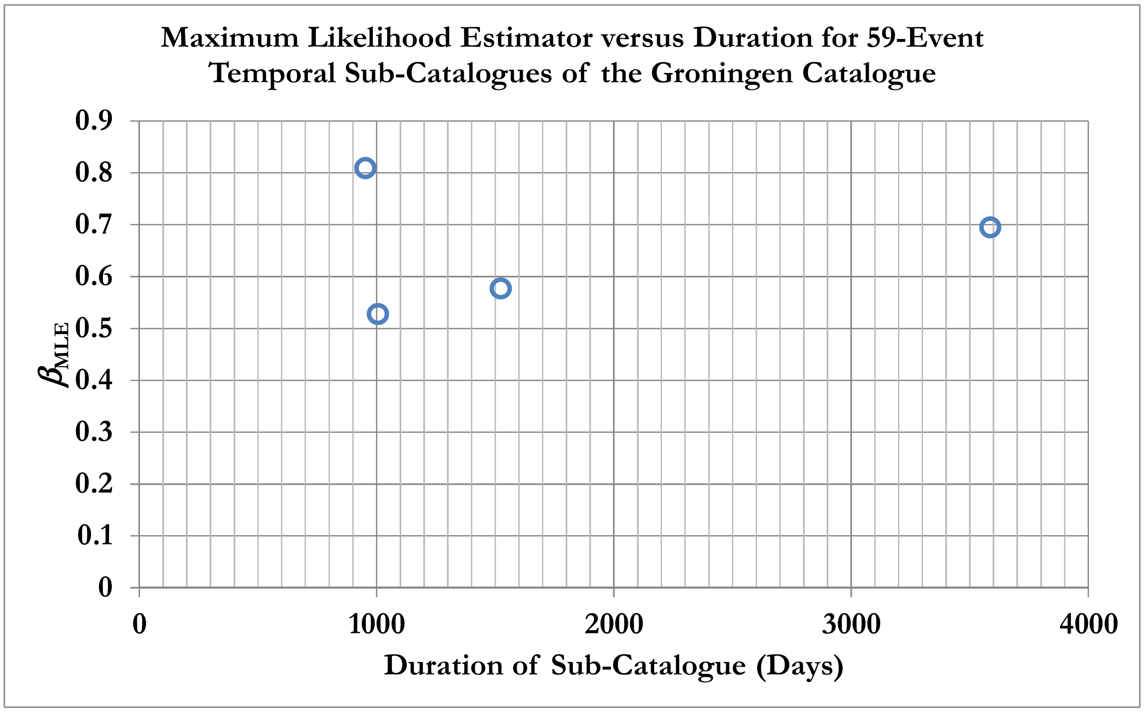


Figure 3.5. Maximum Likelihood Estimators of b-Value for Four 59-Event Sub-Catalogues of the Groningen Catalogue versus Sub-Catalogue Duration.

The maximum likelihood estimate has been computed, according to Eq. (2.19), for each of the sub-catalogues in Table 3.4, and these have been plotted against the duration of the cub-catalogue in Figure 3.6. For these sub-catalogues containing almost equal number of events (29 or 30), it is now clear that there is little correlation between the maximum likelihood estimate of the parameter β and the duration of the sub-catalogue. The error bars (corresponding to ± one standard deviation) shown in Figure 3.6 are obtained from simulations of 1000 realizations of each sub-catalogue with the β for the underlying process set equal to β_{MLE}.

Table 3.4. Distribution of Events in Seismic Moment Bins for the 236-Event Groningen Catalogue (1st May 1995-31st December 2014, Rounded Moment Magnitude ≥ 1.5), Split Up into Eight 29 or 30 Event Temporal Sub-catalogues

Bin Number	1	2	3	4	5	6	7	8
Moment Magnitude	1.5	1.6	1.7	1.8	1.9	2	2.1	2.2
# Events 1*a* Subcatalogue	4	9	2	5	1	3	1	0
# Events 1*b* Subcatalogue	7	7	4	1	2	0	1	2
# Events 2*a* Subcatalogue	7	5	2	3	3	1	0	2
# Events 2*b* Subcatalogue	6	1	5	2	2	2	2	3
# Events 3*a* Subcatalogue	9	5	4	3	0	2	0	0
# Events 3*b* Subcatalogue	7	5	4	4	5	1	0	1
# Events 4*a* Subcatalogue	7	1	3	4	2	4	0	0
# Events 4*b* Subcatalogue	7	5	2	1	3	2	3	0
Bin Number	9	10	11	12	13	14	15	16
Moment Magnitude	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3
# Events 1*a* Subcatalogue	0	1	2	1	0	0	0	0
# Events 1*b* Subcatalogue	1	1	0	0	2	0	0	2
# Events 2*a* Subcatalogue	3	1	1	0	0	0	0	0
# Events 2*b* Subcatalogue	1	0	1	3	0	0	0	1
# Events 3*a* Subcatalogue	2	2	2	0	0	0	0	0
# Events 3*b* Subcatalogue	0	0	2	0	0	0	0	0
# Events 4*a* Subcatalogue	0	3	0	0	2	0	0	1
# Events 4*b* Subcatalogue	1	0	0	1	0	3	1	1
Bin Number	17	18	19	20	21	22		
Moment Magnitude	3.1	3.2	3.3	3.4	3.5	3.6+		
# Events 1*a* Subcatalogue	0	0	0	0	0	0		
# Events 1*b* Subcatalogue	0	0	0	0	0	0		
# Events 2*a* Subcatalogue	0	0	0	0	1	0		
# Events 2*b* Subcatalogue	0	1	0	0	0	0		
# Events 3*a* Subcatalogue	0	0	0	0	0	0		
# Events 3*b* Subcatalogue	0	1	0	0	0	0		
# Events 4*a* Subcatalogue	0	1	0	0	0	1		
# Events 4*b* Subcatalogue	0	0	0	0	0	0		

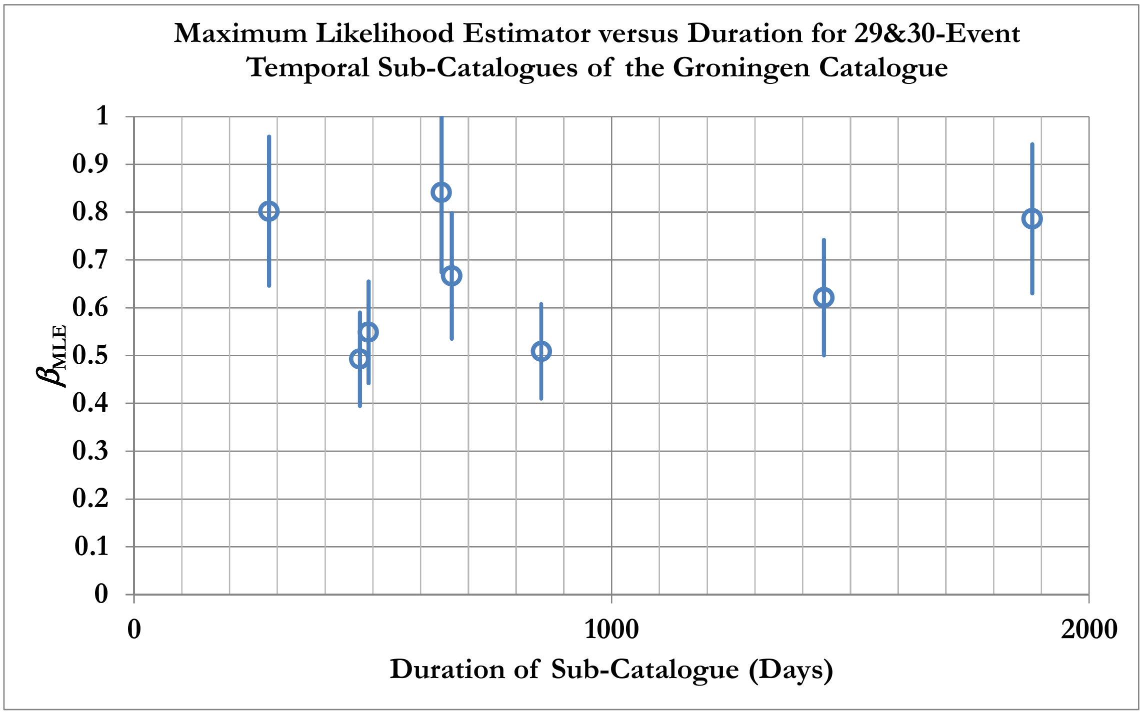


Figure 3.6. Maximum Likelihood Estimators of b-Value for Eight 29 or 30-Event Sub-Catalogues of the Groningen Catalogue versus Sub-Catalogue Duration.

Further ways of temporally subdividing the Groningen catalogue into equal or nearly equal subcatalogues are listed in Table 3.5, and the resulting maximum likelihood estimators of β are shown in Figure 3.7. These results are compared with the hypothesis that the underlying value of β for each sub-catalogue is in fact equal to 2/3 (corresponding to b = 1) in Figure 3.8. The horizontal lines represent the limits of 95% confidence intervals for this hypothesis, obtained from simulations containing 5000 realizations.

Table 3.5. Subdivision of the 256-Event Groningen Catalogue into up to Eight Sub-Catalogues

Number of Sub-Catalogues	**Subdivision**
1	1 × 236
2	2 × 118
3	79, 78, 79
4	4 × 59
5	4 × 47, 48
6	40, 4 × 39, 40
7	33, 5 × 34, 33
8	29, 30, 29, 30, 29, 30, 29, 30

We see that the computed values of β_{MLE} are consistent with the hypothesis that β = 2/3 at the 95% level for all but four of the sub-catalogues, and these four are on the lower bound of the confidence interval rather than being significantly below it.

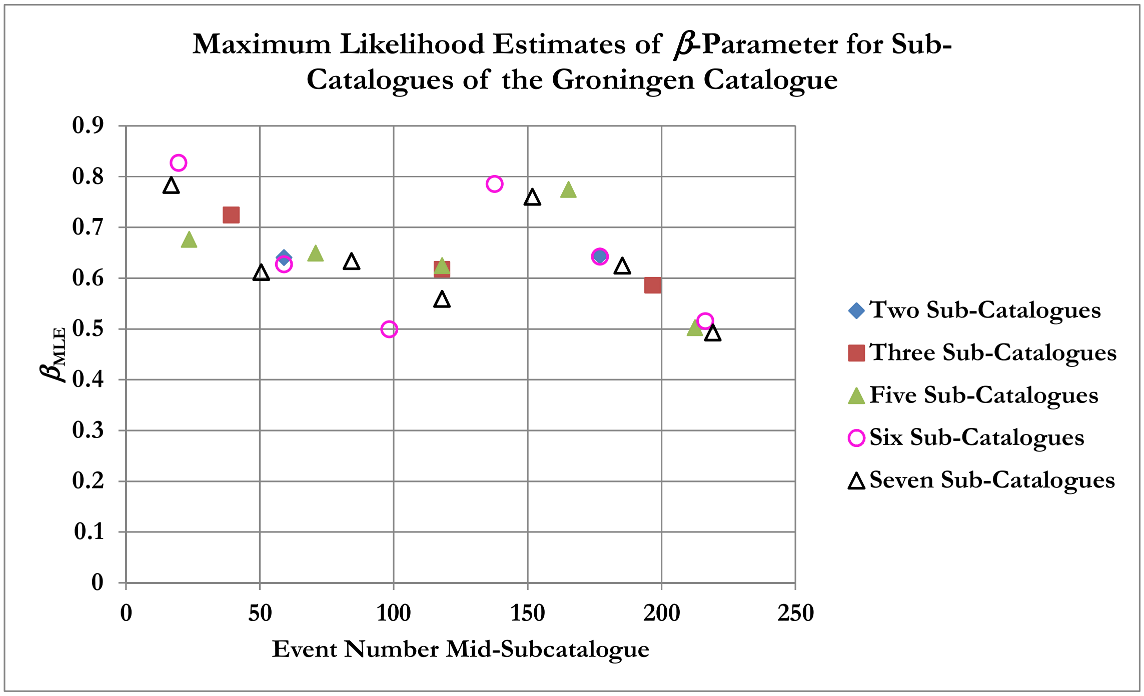


Figure 3.7. *Maximum Likelihood Estimators of β for Sub-Catalogues of the 256-Event Groningen Catalogue Selected as Shown in Table 3.5.*

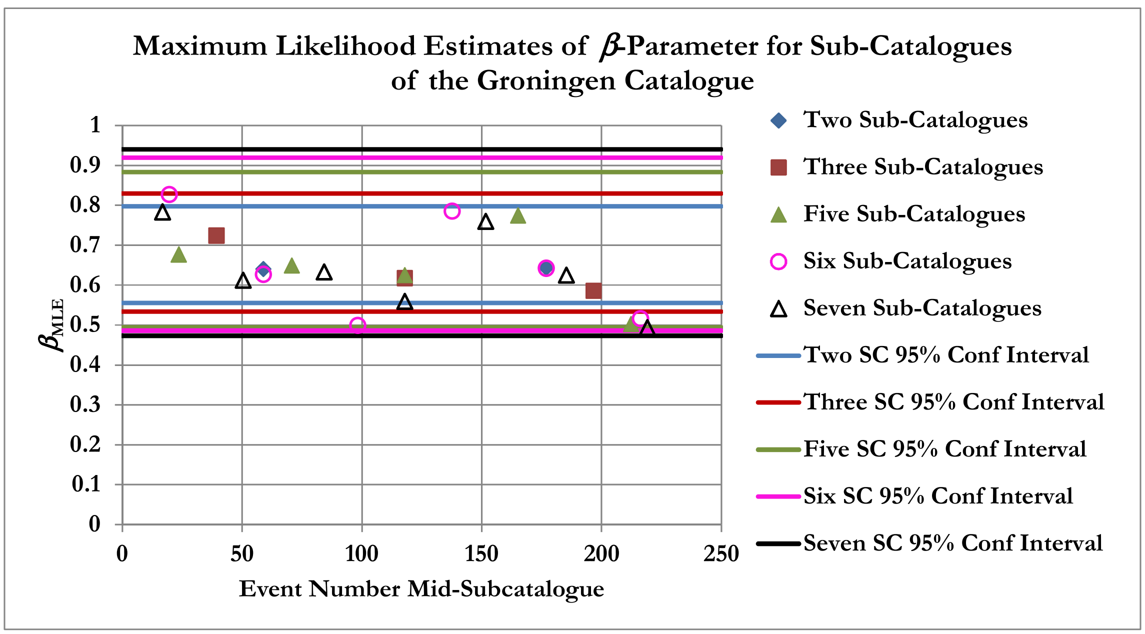


Figure 3.8. *As Figure 3.7 but also showing the 95% Confidence Intervals for the Hypothesis that $\beta = 2/3$ for Each Sub-Catalogue. Note that the Smaller the Sub-Catalogue the Wider the Confidence Interval.*

3.3 Maximum Likelihood and Error Estimates for the b-Value of Events in the Loppersum and Ten Boer Areas

We now focus on two specific spatial sub-regions of the Groningen field centered around Loppersum (containing the region of historically the most intense seismicity, and also the event of maximum moment magnitude) and Ten Boer, the most active region outside Loppersum, and also closer to the city of Groningen. These regions have centres (244, 598) and (250, 591) respectively, in terms of Easting and Northing coordinates expressed in kilometers. The radius of each region is 5 kilometers. These regions correspond to the ones chosen in [5], and the estimates of *b*-value obtained here can be compared with those obtained in [5]. The number of events of magnitude 1.5 or greater in each region during the period 1st May 1995 to 31st December 2014 is 82 for Loppersum and 60 for Ten Boer. The events are tabulated by moment magnitude in Table 3.4. The spatial location of these events, as well as an indication of magnitude, is shown in Figure 3.5.

Table 3.6. Distribution of Events in Seismic Moment Bins for the Loppersum and Ten Boer regional sub-catalogues (1st May 1995-31st December 2014, Rounded Moment Magnitude ≥ 1.5).

Bin Number	1	2	3	4	5	6	7	8
Moment Magnitude	1.5	1.6	1.7	1.8	1.9	2	2.1	2.2
Loppersum Region	14	8	7	6	7	8	2	6
Ten Boer Region	15	10	11	4	4	4	0	1
Bin Number	9	10	11	12	13	14	15	16
Moment Magnitude	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3
Loppersum Region	4	2	3	2	4	1	1	3
Ten Boer Region	1	4	2	1	0	0	0	2
Bin Number	17	18	19	20	21	22		
Moment Magnitude	3.1	3.2	3.3	3.4	3.5	3.6+		
Loppersum Region	0	2	0	0	1	1		
Ten Boer Region	0	1	0	0	0	0		

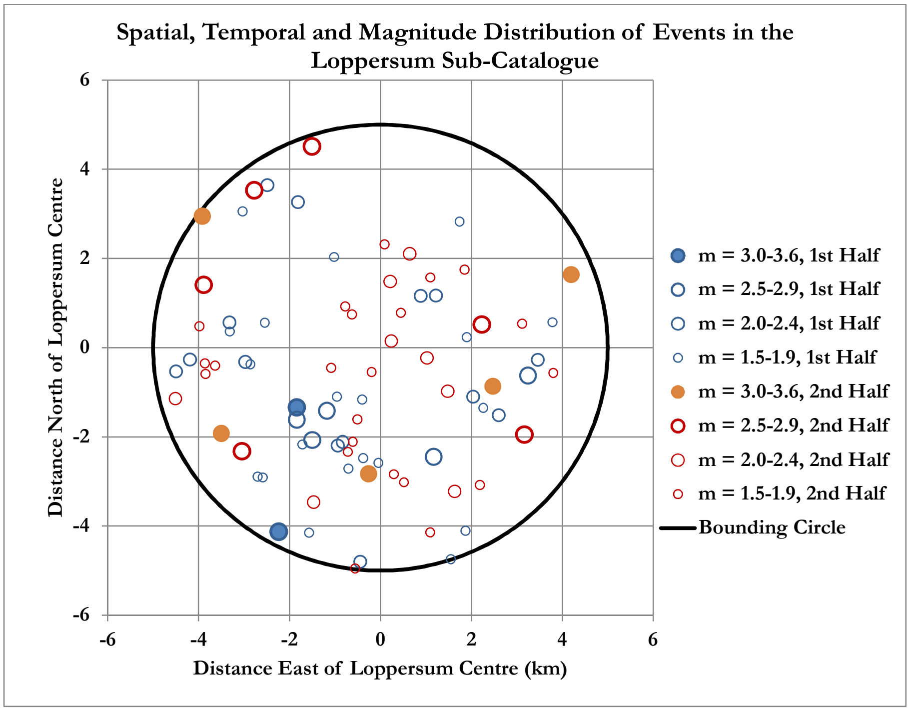
Note that the events are color-coded according to whether they occur in the first half (by time of event) or second half of the sub-catalogue. These two halves are considered separately in the next section.

Applying Eq. (2.18) to the data in Table 3.5, we obtain the following values of β_{MLE} for the Loppersum and Ten Boer sub-catalogues (the corresponding values of b_{MLE} are given in brackets):

Loppersum $\beta_{MLE} = 0.471$ ($b_{MLE} = 0.707$) (3.7)

Ten Boer $\beta_{MLE} = 0.709$ ($b_{MLE} = 1.064$) (3.8)

1000 realizations of the 82-event Loppersum sub-catalogue and the 60-event Ten Boer sub-catalogue with input value $\beta = 2/3$ yielded $\beta_{MLE} = 0.668 \pm 0.074$ for Loppersum and 0.669 ± 0.085 for Ten Boer. The actual value of β_{MLE} is far outside the error bars of the simulation for Loppersum and well within it for Ten Boer. Further illustration of this is provided by Figure 3.6 which shows the distribution of β_{MLE} for the two simulations together with the actual values in Eqs. (3.7) and (3.8). The shapes of the distributions are compared with Gaussian distributions with the same mean and variance for comparison. Figure 3.6 (a), in particular, tells us that we can be 99.9% confident that the underlying process producing induced seismicity in the Loppersum region has a β value smaller than 2/3. Note that the estimate of β obtained by the present method for the Loppersum sub-region is smaller than that obtained in [5] using a straight line fit of log frequency versus seismic moment. The value obtained there was $\beta = 0.57$, corresponding to $b = 0.85$. This method for estimating b-value has been used in the past, but as pointed out in [6], is not justified on statistical grounds.



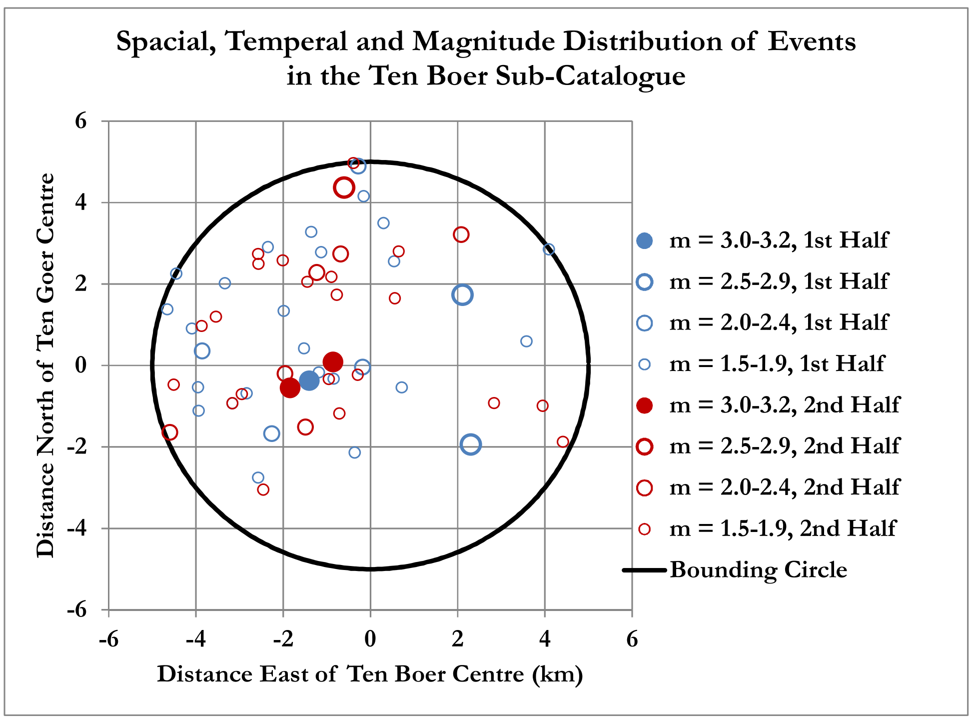
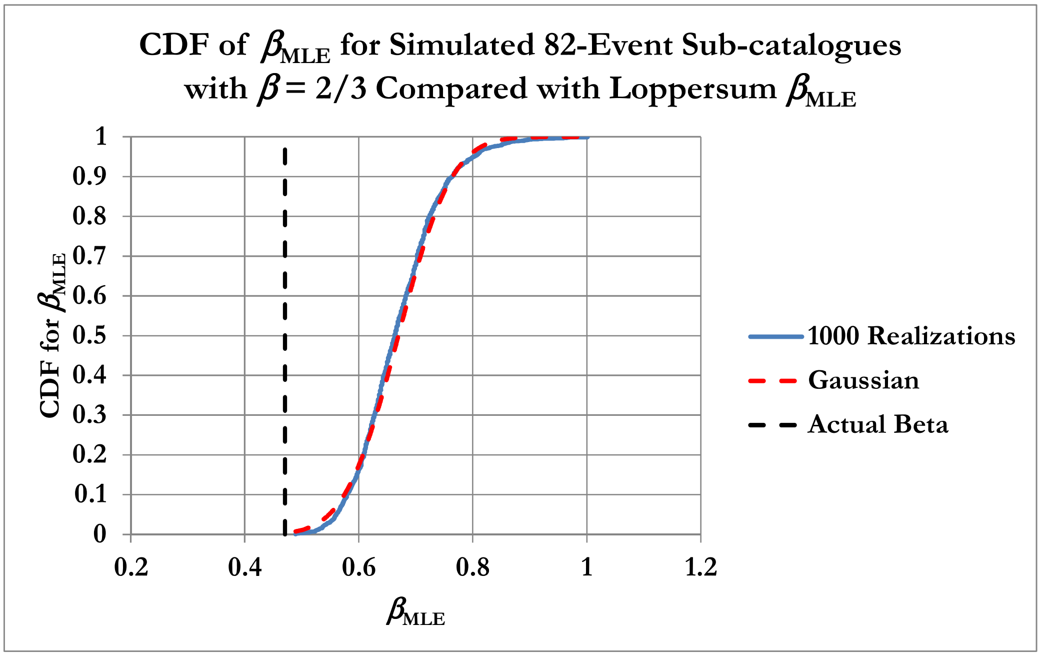


Figure 3.9. *Spatial distribution of events in (a) the Loppersum and (b) Ten Boer regions. Magnitude classes are also indicated . The terms 1st Half and 2nd Half refer to the first and second halves of the catalogues, arranged temporally*



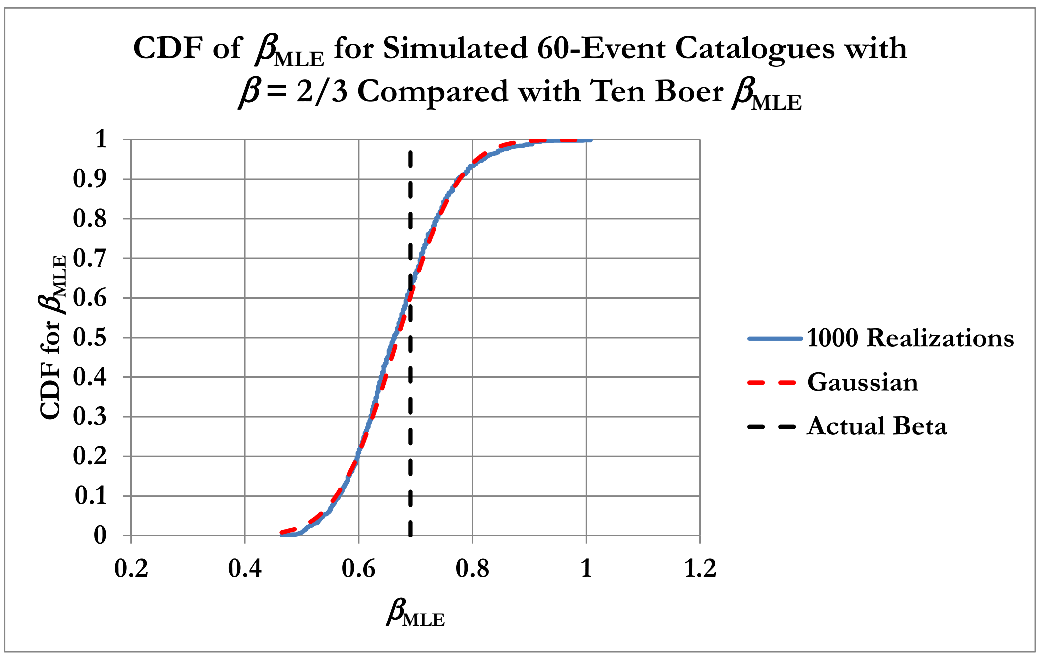


Figure 3.10. *Distribution of maximum likelihood estimate* β_{MLE} *for 1000 realizations of simulated (a) Loppersum and (b) Ten Boer sub-catalogues with* $\beta = 2/3$. *The shapes of the distributions are compared with Gaussian distributions having the same mean and variance and the values of* β_{MLE} *obtained for the actual sub-catalogues are also shown.*

3.4 Maximum Likelihood and Error Estimates for the *b*-Value of Temporal Subsets of Events in the Loppersum and Ten Boer Areas

Because the number of events in the Loppersum and Ten Boer sub-regions is relatively small, just two temporal subsets, corresponding to the first and second halves of the sub-catalogue corresponding to each sub-region, are used. The distribution of the events into the magnitude bins for each temporal subset is shown in Table 3.6.

Table 3.7. Distribution of Events in Seismic Moment Bins for the first and second halves of the Loppersum and Ten Boer subcatalogues (1st May 1995-31st December 2014, Rounded Moment Magnitude ≥ 1.5).

Bin Number	1	2	3	4	5	6	7	8
Moment Magnitude	1.5	1.6	1.7	1.8	1.9	2	2.1	2.2
Loppersum Region (1st Half)	7	7	1	2	3	5	1	2
Loppersum Region (2nd Half)	7	1	6	4	4	3	1	4
Ten Boer Region (1st Half)	7	7	6	1	2	1	0	1
Ten Boer Region (2nd Half)	8	3	5	3	2	3	0	0
Bin Number	9	10	11	12	13	14	15	16
Moment Magnitude	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3
Loppersum Region (1st Half)	4	2	2	1	2	0	0	1
Loppersum Region (2nd Half)	0	0	1	1	2	1	1	2
Ten Boer Region (1st Half)	0	2	1	1	0	0	0	1
Ten Boer Region (2nd Half)	1	2	1	0	0	0	0	1
Bin Number	17	18	19	20	21	22		
Moment Magnitude	3.1	3.2	3.3	3.4	3.5	3.6+		
Loppersum Region (1st Half)	0	0	0	0	1	0		
Loppersum Region (2nd Half)	0	2	0	0	0	1		
Ten Boer Region (1st Half)	0	1	0	0	0	0		
Ten Boer Region (2nd Half)	0	0	0	0	0	0		

The values of β_{MLE} for each temporal sub-catalogue, computed using Eq. (2.19), are listed in Eqs. (3.9a)-(3.9d), with the corresponding values of b_{MLE} in brackets.

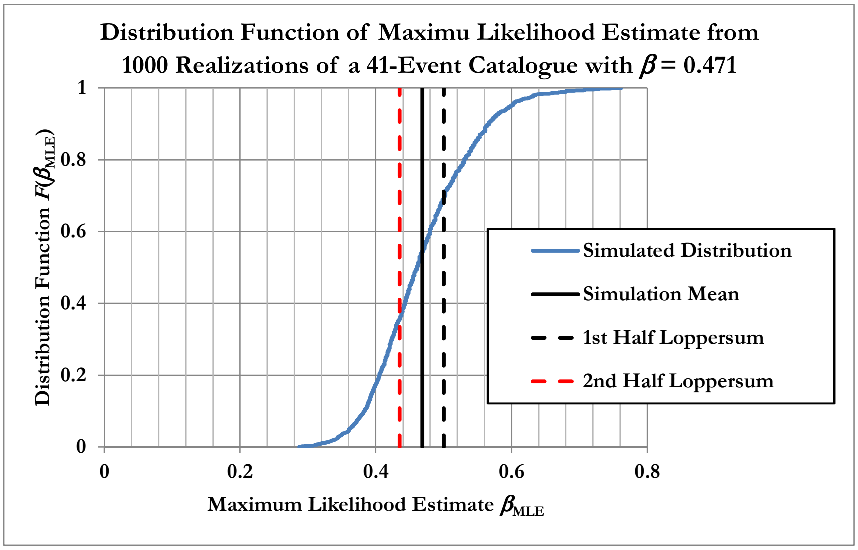
Loppersum, 1st half $$\beta_{MLE} = 0.500 \ (b_{MLE} = 0.751) \tag{3.9a}$$

Loppersum, 2nd half $$\beta_{MLE} = 0.435 \ (b_{MLE} = 0.652) \tag{3.9b}$$

Ten Boer, 1st half $\beta_{MLE} = 0.764 \ (b_{MLE} = 1.146)$ (3.9c)

Ten Boer, 2nd half $\beta_{MLE} = 0.661 \ (b_{MLE} = 0.992)$ (3.9d)

In order to test whether there is a significant temporal shift in the values of β_{MLE} for the Loppersum and Ten Boer sub-regions, simulations of 41 and 30 event catalogues were performed, these being the number of events in the Loppersum and Ten Boer temporal sub-catalogues, respectively. The values of β used as input were the maximum likelihood estimates of the entire catalogue for each subregion, namely the values given in Eqs. (3.7) and (3.8) respectively. The results are shown in Figure 3.7.



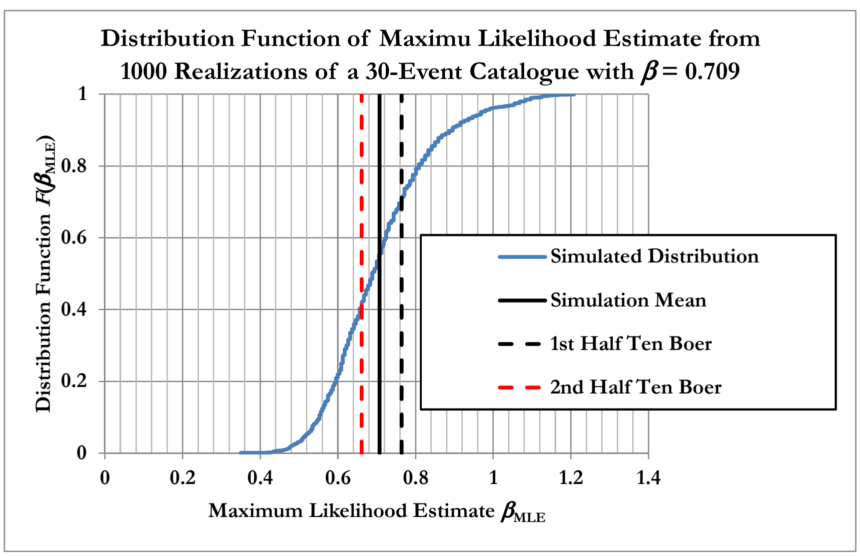


Figure 3.9. Distribution of maximum likelihood estimate β_{MLE} for 1000 realizations of simulated (a) Loppersum and (b) Ten Boer sub-catalogues with $\beta = 2/3$. The shapes of the distributions are compared with Gaussian distributions having the same mean and variance and the values of β_{MLE} obtained for the actual sub-catalogues are also shown.

We can conclude from Figure 3.6 that the differences in the β_{MLE} observed between the first and second halves of the catalogues for Loppersum and Ten Boer are not significant enough to indicate a change in the underlying process.

4. Conclusions and Recommendations

This report contains a prescription for obtaining a maximum likelihood estimate for the *b*-value in the Gutenberg-Richter relationship for the frequency versus moment magnitude for seismic events. This relationship has been applied for both naturally occurring and induced seismicity. A procedure for obtaining the error estimate in the resulting values of *b* is also outlined.

The prescription and procedure is then applied to the catalogue of seismic events triggered by gas production in the Groningen field. Events occurring between 1st May 1995 and 31st December 2014, and of magnitude 1.5 and greater are used to estimate values of *b* for the entire catalogue and various subsets of it. The value of *b* obtained for the entire catalogue is consistent with the commonly accepted value of unity for induced seismicity.

A key question for hazard assessment is whether the character of the seismicity is changing with time, or is dependent on the number of events per unit time. This is because the frequency of rare events of high magnitude, potentially capable of causing structural damage, is dependent on the *b*-value. Accordingly, the Groningen catalogue is then applied to temporal subsets to examine any evidence for a varying *b*-value. While a subdivision of the catalogue into four subsets of equal numbers of events suggests more variability in the maximum likelihood estimator for the subsets than would be expected if the *b*-value of the underlying process were constant, a closer examination, involving further subdivisions of the catalogue, does not show any systematic dependence of *b* value on event rate.

A recent report examined induced seismicity in various spatial sub-regions of the Groningen field and beyond it and related the data to a stochastic model of fault-slippage induced seismicity. Two of these sub-regions, around Loppersum, where the event of largest magnitude to date has occurred, and around Ten Boer, which is closer to the city of Groningen, were selected for analysis by the methods introduced in this report. All events of magnitude greater or equal to 1.5 and occurring between 1st May 1995 and 31st December 2014 were considered. The results showed that the *b*-value around Loppersum is less than unity to a high degree of significance, while the Ten Boer data is consistent with a *b*-value of unity. No statistically significant temporal evolution of *b*-value for these spatial sub-regions was found. Note that no attempt to obtain a spatial correlation dimension for events, such as was done for seismicity in the Lacq gas field [7], was made here.

Though it resembles the one introduced in [7], the method presented here for determining *b*-value is different from any proposed in the literature and it should be applied to literature datasets in order to compare with these other methods. Furthermore, the *b*-value estimator is largely dependent on a single parameter, the summation of the number of events occurring in a magnitude moment bin times the bin number. Examination of the magnitude of the likelihood function itself should be performed in order to identify data anomalies or inconsistencies.

5. Nomenclature

Roman Characters

Symbol	Definition	Unit	Introduced
b	b-value in Gutenberg-Richter law	-	Page II
b_{MLE}	Maximum likelihood estimator of b	-	Page II
d	Constant relating moment and moment magnitude	-	Eq. (1.1)
$F(M)$	Distribution function for seismic moment	-	Eq. (1.6)
F_C	Complementary distribution function	-	Eq. (2.6)
$f(r_1..r_n)$	Arbitrary function of bin event numbers	-	Eq. (2.20)
$L(p_1..p_n)$	Likelihood function	-	Eq. (2.10)
M	Seismic moment	N-m	Eq. (1.1)
M_0	Threshold seismic moment	N-m	Eq. (2.1)
M_c	Reference seismic moment	N-m	Eq. (1.1)
m	Moment magnitude	-	II
m_0	Threshold moment magnitude	-	II
N	Number of events in a catalogue	-	Eq. (2.8)
n	Largest bin number containing an event	-	Eq. (2.5)
$P(m)$	Probability that an event has moment magnitude	-	II
$P_N(m)$	Probability that the largest event in an N-event catalogue has moment magnitude > m	-	Eq. (3.6)
$P(r_1..r_n)$	Probability that r_k events fall into the kth bin for all bins 1 to n	-	Eq. (2.8)
p_k	Probability that an event falls in the kth bin	-	Eq. (2.5)
r_k	Number of events in the kth bin	-	Eq. (2.8)
S_0	Component of maximum likelihood estimator	-	Eq. (A.2)
S_1	Component of maximum likelihood estimator	-	Eq. (A.2)
$S_1(\varepsilon)$	Generalization of S_1	-	Eq. (A.9)
u	Uniformly distributed random variable over [0, 1]	-	Eq. (2.23)
x	Normalized seismic moment (M/M_0)	-	Eq. (2.3)

Greek Characters

Symbol	Definition	Unit	Introduced
α	Ratio of moments for a moment magnitude increment of 0.1	-	Eq. (2.4)
β	Exponent in Pareto distribution of nromalized moment	-	Eq. (1.7)
β_{MLE}	Maximum likelihood estimator of β	-	Eq. (2.19)
ε	Small quantity set to zero at end of calculation	-	Eq. (A.9)
μ_{MLE}	Mean of distribution of β_{MLE}	-	Eq. (2.20)
σ_{MLE}	Variance of distribution of β_{MLE}	-	Eq. (2.21)

References

1. Gutenberg B. & Richter, C.F., (1954), Seismicity of the Earth and Associated Phenomena. 2nd Edition, Princeton University, Princeton, New Jersey.
2. Grasso, J-R & Sornette, D., (1998), Testing Self-Organized Criticality by Induced Seismicity. J. Geophysical Research 103 (B12), 29965-29987.
3. Bourne, S. J., and Oates, S., (2012), Induced Strain and Induced Earthquakes within the Groningen Gas Field: Earthquake Probability Estimates associated with Future Gas Production" Technical Report SR.13.xxxxx, Shell Global Solutions International BV, 2013.
4. Hanks, T., and H. Kanamori (1979), Moment Magnitude Scale, J. Geophysical Research 84, 2348-2350.
5. Wentinck, H.M., (2015), Seismic Induced Activity – Statistical Geomechanical Assessments of Tremors along Faults in a Compacting Reservoir. Technical Report SR.15.xxxxx, Shell Global Solutions International BV, 2015.
6. Marzocchi, W, & Sandri, L., (2003), A Review and New Insights on Estimation of the *b*-Value and its Uncertainty. Annals of Geophysics 46 (6), 71-79.
7. Lahaie, F. & Grasso, J.R., (1999), Load Rate Impact on Fracturing Pattern: Lessons from Hydrocarbon Recovery, Lacq Gas Field, France. J. Geophysical Research 104 (B8), 17941-17954.
8. Bender, B., (1983), Maximum Likelihood Estimation of *b* Value for Magnitude Grouped Data. Bull. Seismological Society of America 73 (3) 831-851.

Appendix 1 – Conversion of the Summations in Eqs. (2.20) & (2.22) into Integrations

In this appendix a methodology for converting the summations on the right-hand sides of Eqs. (2.20) and (2.22) into integrations over a single variable is outlined. Considering Eq. (2.19) first of all, we write

$$\beta_{MLE} = \{S_0 - S_1\}/\ln(\alpha) \tag{A.1}$$

where $S_0 \equiv \ln\left\{\sum_{k=1}^{n} kr_k - r_n\right\}; S_1 \equiv \ln\left\{\sum_{k=1}^{n}(k-1)r_k\right\}$ (A.2)

The conversion is accomplished by using the following integral representation of the logarithm:

$$\ln(z) = \int_0^\infty du u^{-1}\{\exp(-u) - \exp(-zu)\} \tag{A.3}$$

Hence $S_0 = \int_0^\infty du\left\{\exp(-u) - \exp\left\{-\left(\sum_{k=1}^{n} kr_k - r_n\right)\right\}u\right\}/u$ (A.4)

The unrestricted summation over S_0 may now be evaluated explicitly on reversing the order of summation and integration to yield

$$< S_0 >= \int_0^\infty du u^{-1}\left\{\exp(-u) - (\sum_{k=1}^{n-1} e^{-uk} p_k + e^{-u(n-1)} p_n)^N\right\} \tag{A.5}$$

The restricted sum may readily be related to the unrestricted one by subtracting the term $\ln(N)p_1^N$ and dividing by $1 - p_1^N$. The result is

$$< S_0 >' = \{< S_0 > - \ln(N)p_1^N\}/(1 - p_1^N) \tag{A.6}$$

Integrating Eq. (A.5) by parts, we obtain

$$< S_0 >= \int_0^\infty du \ln(u)\left\{\exp(-u) + \frac{d}{du}(\sum_{k=1}^{n-1} e^{-uk} p_k + e^{-u(n-1)} p_n)^N\right\} \tag{A.7}$$

So that we finally have

$$< S_0 >' = \frac{1 - \ln(N)p_1^N}{1 - p_1^N}\int_0^\infty du \ln(u)\left\{\exp(-u) + \frac{d}{du}(\sum_{k=1}^{n-1} e^{-uk} p_k + e^{-u(n-1)} p_n)^N\right\} \tag{A.8}$$

We proceed in a slightly different way for S_1 due to the fact that $<S_1>$ is infinite. We introduce the quantity

$$S_1(\varepsilon) \equiv \ln\left\{\varepsilon + \sum_{k=1}^{n}(k-1)r_k\right\} \tag{A.9}$$

where ε is a positive quantity that we set to zero at the end of the calculation. We now have

$$< S_1(\varepsilon) >' = \left\{< S_1(\varepsilon) > - \ln(\varepsilon)\, p_1{}^N\right\}/(1 - p_1{}^N) \tag{A.10}$$

Using Eq. (A.3), we have

$$S_1(\varepsilon) = \int_0^\infty du \left\{\exp(-u) - \exp\left\{-\left(\varepsilon + \sum_{k=1}^{n}(k-1)r_k\right)\right\}u\right\}/u \tag{A.11}$$

Interchanging the order of integration and summation in the unrestricted sum, and evaluating the sum yields

$$< S_1(\varepsilon) > = \int_0^\infty du u^{-1}\left\{\exp(-u) - \exp(-\varepsilon u)\left(\sum_{k=1}^{n} p_n \exp\{-(k-1)u\}\right)^N\right\} \tag{A.12}$$

Since $\ln(\varepsilon) = \int_0^\infty du u^{-1}\{\exp(-u) - \exp(-\varepsilon u)\}$ (A.13)

we may write

$$< S_1(\varepsilon) >' = \int_0^\infty du u^{-1}\left\{\exp(-u) - \exp(-\varepsilon u)\left\{\left(\sum_{k=1}^{n} p_k \exp\{-(k-1)u\}\right)^N - p_1{}^N\right\}/(1 - p_1{}^N)\right\} \tag{A.14}$$

Integration by parts yields the result

$$< S_1(\varepsilon) >' = \int_0^\infty du \ln(u)\left\{\exp(-u) + \exp(-\varepsilon u)\frac{d}{du}\left(\sum_{k=1}^{n} p_k \exp\{-(k-1)u\}\right)^N /(1 - p_1{}^N)\right\}$$
$$- \varepsilon\int_0^\infty du \ln(u)\exp(-\varepsilon u)\left\{\left(\sum_{k=1}^{n} p_k \exp\{-(k-1)u\}\right)^N - p_1{}^N\right\}/(1 - p_1{}^N) \tag{A.15}$$

As $\varepsilon \to 0$ the first term on the right-hand side of Eq. (A.15) tends to a constant and the second term tends to zero as ε. Hence,

$$< S_1 >' = < S_1(0) >' = \int_0^\infty du \ln(u)\left\{(1 - p_1{}^N)\exp(-u) + \frac{d}{du}\left(\sum_{k=1}^{n} p_k \exp\{-(k-1)u\}\right)^N\right\} \tag{A.16}$$

For numerical integration purposes, or to develop asymptotic expansions for large N, it is useful to transform to the new variables v_1 and v_2 in Eqs. (94) and (95) respectively as follows:

$$\exp(-v_1) = \sum_{k=1}^{n-1} e^{-uk} p_k + e^{-u(n-1)} p_n \tag{A.17}$$

and
$$\exp(-v_2) = \sum_{k=1}^{n-1} e^{-u(k-1+\varepsilon)} p_k \tag{A.18}$$

Raising Eqs. (A.17) and (A.18) to the power N and differentiating, we obtain

$$dv_1 N \exp(-v_1 N) = -\frac{d}{du}(\sum_{k=1}^{n-1} e^{-uk} p_k + e^{-u(n-1)} p_n)^N du \tag{A.19}$$

and
$$dv_2 N \exp(-v_2 N) = -\frac{d}{du}(\sum_{k=1}^{n} e^{-u(k-1+\varepsilon)} p_k)^N du \tag{A.20}$$

On substituting Eqs. (98) and (99) back into Eqs. (94) and (95), we get

$$< S_0 >' = \frac{1-\ln(N) p_1{}^N}{1-p_1{}^N} \left\{ \int_0^\infty du \ln(u) \exp(-u) - N \int_0^\infty dv_1 \ln(u) \exp(-N v_1) \right\} \tag{A.21}$$

and
$$< S_1 >' = \int_0^\infty du \ln(u) \exp(-u) - N \int_0^\infty dv_2 \ln(u) \exp(-N v_2) / (1 - p_1{}^N) \tag{A.22}$$

Developing expansions for u in powers of v_1 in Eq. (A.21) and v_2 in Eq. (A.22) then leads to large N asymptotic expansions for $<S_0>'$ and $<S_1>'$.

We shall not deal in detail here with the representation of $<\beta_{\mathrm{MLE}}{}^2>'$ as an integral, only mentioning that we make use of the fact that (*cf* Eq. A.3)

$$\ln(z)\ln(z') = \int_0^\infty du u^{-1} \int_0^\infty dv v^{-1} \{\exp(-u) - \exp(-zu)\}\{\exp(-v) - \exp(-z'v)\} \tag{A.23}$$

Bibliographic information

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